

COMP 4045 – Assignment 4

1. **Diameter.** Let P be a set of n points in \mathcal{R}^d and let ϵ be a positive constant. The diameter of P is the distance between the two points in P furthest away from each other.

Present an algorithm that approximates the diameter of P within a factor of $(1 + \epsilon)$. The running time of the algorithms should be $O(n \log n + s^d \cdot n)$, for some constant s . Don't forget to prove the correctness and the time complexity of the algorithm.

2. **Closest pair distance.** Let P be a set of n points in \mathcal{R}^d . Compute the distance between the pair of points in P that are closest.

Below is an algorithm that computes the distance between the closest pair. Prove correctness and time complexity.

Algorithm CLOSEST PAIR(P)

1. $\{A_1, B_1\}, \dots, \{A_m, B_m\} \leftarrow$ well-separated pair decomposition of P with $s = 4$.
2. distance $\leftarrow \infty$
3. **for** $i \leftarrow 1$ **to** m **do**
4. **if** $\#A_i = 1$ and $\#B_i = 1$ **then**
5. Set a and b to be the point in A_i and B_i , respectively.
6. **if** $|ab| < \text{distance}$ **then**
7. distance $\leftarrow |ab|$
8. **return** $G' = (V, E')$.

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3. It is possible to reduce the size of a quadtree of depth d for a set of points (with real coordinates) inside a square from $O((d+1)n)$ to $O(n)$. The idea is to discard any node v that has only one child under which points are stored. The node is discarded by replacing the pointer from the parent of v to v by the pointer from the parent to the only interesting child of v . Prove that the resulting tree has linear size. Can you also improve upon the $O((d+1)n)$ construction time? (Exercises 14.5)
 4. Quadtrees can be used to perform range queries. Describe an algorithm for querying a quadtree on a set P of points with a query region R . Analyze the worst-case query time for the case where R is a rectangle, and for the case where R is a half-plane bounded by a vertical line. (Exercises 14.13)

Due: 5pm, 9 June 2005 (please submit the assignment to kai.xu@nicta.com.au or to my office at IMAGEN group, NICTA, Bay 15, Australia Technology Park (next to Redfern station))