

## COMP 4045 – Assignment 2

1. A simple (hole-free) polygon  $P$  is called *star-shaped* if it contains a point  $q$  such that for any point  $p$  in  $P$  the line segment  $\overline{pq}$  is contained in  $P$ , see Fig. 1. Give an  $O(n \log n)$  time algorithm, or an  $O(n)$  randomised expected time algorithm, that decides if a given polygon is star-shaped or not.
2. Exercise 5.3 in the course book.
3. Exercise 5.9 in the course book.
4. Exercise 5.10(a-b) in the course book.
5. *Point queries among rectangles.* Let  $P$  be a set of  $n$  rectangles in the plane. Report all rectangles in  $P$  that intersects a query point  $q$ . Describe a data structure for this problem that uses  $O(n \log n)$  preprocessing,  $O(n)$  storage and  $O(n^{3/4} + k)$  query time. *Hint:* How can a rectangle in 2D be described in 4D? (How can a rectangle in 1D be described in 2D?)
6. Give an algorithm that computes in  $O(n \log n)$  time a diagonal that splits a simple polygon with  $n$  vertices into two simple polygons each with at most  $\lfloor 2n/3 \rfloor + 2$  vertices. *Hint:* Use the dual graph of a triangulation.

**Bonus:** Prove that any polygon  $P$  with  $n$  vertices admits a triangulation, even if it has  $h$  polygonal holes. Prove that the number of triangles in the triangulation is  $n + 2h - 2$ .

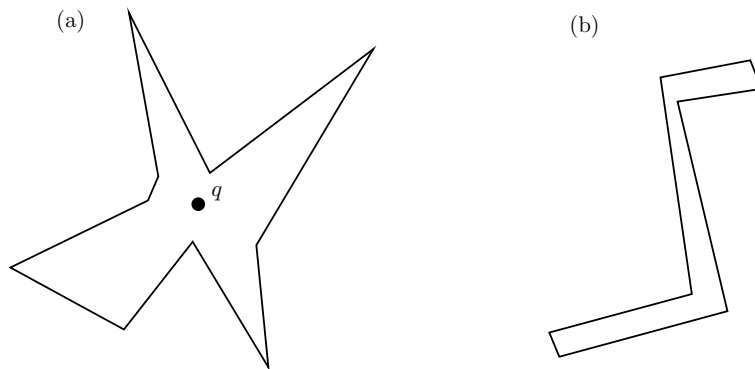


Figure 1: (a) A star-shaped polygon since  $p$  can "see" all of  $P$ . (b) An example of a simple polygon that is not star-shaped.

**Due: 28th of April 2005 at 4pm.**