1. A simple (hole-free) polygon $P$ is called star-shaped if it contains a point $q$ such that for any point $p$ in $P$ the line segment $pq$ is contained in $P$, see Fig. 1. Give an $O(n \log n)$ time algorithm, or an $O(n)$ randomised expected time algorithm, that decides if a given polygon is star-shaped or not.

2. Exercise 5.3 in the course book.

3. Exercise 5.9 in the course book.

4. Exercise 5.10(a-b) in the course book.

5. Point queries among rectangles. Let $P$ be a set of $n$ rectangles in the plane. Report all rectangles in $P$ that intersects a query point $q$. Describe a data structure for this problem that uses $O(n \log n)$ preprocessing, $O(n)$ storage and $O(n^{3/4} + k)$ query time. 

\textbf{Hint:} How can a rectangle in 2D be described in 4D? (How can a rectangle in 1D be described in 2D?)

6. Give an algorithm that computes in $O(n \log n)$ time a diagonal that splits a simple polygon with $n$ vertices into two simple polygons each with at most $\lfloor 2n/3 \rfloor + 2$ vertices. 

\textbf{Hint:} Use the dual graph of a triangulation.

\textbf{Bonus:} Prove that any polygon $P$ with $n$ vertices admits a triangulation, even if it has $h$ polygonal holes. Prove that the number of triangles in the triangulation is $n + 2h - 2$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Figure1.png}
\caption{(a) A star-shaped polygon since $p$ can "see" all of $P$. (b) An example of a simple polygon that is not star-shaped.}
\end{figure}

Due: 28th of April 2005 at 4pm.