Advanced Review material

(i) Use the natural deduction propositional proof rules listed below, to fill in the justification in the right column in each of the steps of the proof shown for the fact that one can deduce Q \rightarrow S from the premises (P or Q) \rightarrow R, (Q and R) \rightarrow S. Note that we use here notation like “and” and “or” rather than special symbols.

The rules:
- And-intro: From E, F deduce E and F
- And-elim1: From E and F deduce E
- And-elim2: From E and F deduce F
- Or-intro1: From E deduce E or F
- Or-intro2: From F deduce E or F
- Or-elim: From E or F, (boxed deduction of G from E), (boxed deduction of G from F) deduce G
- Imply-intro: From (boxed deduction of F from E) deduce F \rightarrow E
- Imply-elim: From E, E \rightarrow F deduce F
- Not-intro: From (boxed deduction of false from E), deduce not E
- Not-elim: From E, not E deduce false
- False-elim: From false deduce E
- Double-not-elim: From not not E deduce E

\[ (P \lor Q) \rightarrow R \]
\[ P \rightarrow Q \]

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>R</th>
<th>Q and R</th>
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</thead>
<tbody>
<tr>
<td>assumption</td>
<td>premise</td>
<td>premise</td>
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(ii) This part deals with first-order logic, using the following predicates. S(x) means x is a student. D(x) means x is on the Dean’s List. T(y) means y is a test. A(x,y) means x attempted y. P(x,y) means x obtained a pass on y. Express the following English statements in first-order logic. Do not use comprehension notation to restrict the domains of quantified variables in your answers.

(a) Every test has been attempted by some student.
Answer:

(b) Some student has attempted every test.
Answer:

(c) At least one student has passed a test.
Answer:

(d) There is a student who is on the Dean’s List and has attempted a test which he/she hasn’t passed.
Answer:

(iii) Draw the transition system for the following FSP process P1, and show one partial trace of the system which has 5 actions beginning as shown (that is, continue the trace with 3 more actions).

\[ P1 = (a \rightarrow P2 \mid b \rightarrow P3 \mid c \rightarrow STOP), \]
\[ P2 = (a \rightarrow P3 \mid b \rightarrow P4), \]
\[ P3 = (c \rightarrow P1), \]
\[ P4 = (a \rightarrow STOP). \]

The LTS diagram is
One trace is $a \rightarrow a \rightarrow$

(iv) Given the processes defined below, show a partial trace which is 10 actions long, for the process SYS.

$Q_1 = (\text{zig} \rightarrow \text{zag} \rightarrow Q_1)$.
$Q_2 = (\text{biff} \rightarrow Q_3 \mid \text{bong} \rightarrow \text{bang} \rightarrow Q_2)$.
$Q_3 = (\text{baff} \rightarrow Q_2)$.

$||\text{SYS} = (a:Q_1 \parallel b:Q_2)/\text{synch}/\{\text{a.zag}, \text{b.baff}\}$.

One trace is

(v) Given a fact base with information about the predicates $race(\text{Athlete}, \text{Event}, \text{Time})$, $involved(\text{Industry}, \text{Company})$ and $sponsor(\text{Athlete}, \text{Company}, \text{amount})$, write Prolog queries or rules to express the conditions below.

(a) Write a query to find the pairs of companies which have sponsored athletes who participated in the same race.

(b) Write a rule or rules to define a relationship $\text{competitor}(X,Y)$ where we say that $X$ is a competitor of $Y$ if there is some industry in which both are involved.

(c) Write a rule or rules to define a predicate $\text{beats}(X,Y)$, which means that there is some event in which $X$ had a lower time than $Y$.

(d) Write a query to find a situation where there is a chain so that $X_1$ beats $X_2$, $X_2$ beats $X_3$, etc, and $X_n$ beats $X_1$. 