Enterprise-Scale Software Development (COMP5348)

Semester 1, 2009

Tutorial Week 10 Solutions

The first two questions are adapted from Gray-Reuter, ch 3.

Q1. Look at the US mortality rate statistics at
http://www.disastercenter.com/cdc/Death%20rates%202005.html
It states that the mortality rate for people between 20 and 25 years old was about 98 per 100,000 (that is, about 0.1%). Which measure of system failure is this similar to? What is wrong with the following argument: "a 22-year old has about 0.1% chance to die each year; therefore their average life expectancy (the time till they die) is 1000 years"?

Consider the following questions. For each, estimate an appropriate answer, using the data from the web page mentioned

a) In a city with 100,000 22-year-olds, how many of them would be expected to die in a year? How many would be expected to die in a one-week period?
The number "expected" to die within a year is 98. This doesn't mean that it is likely that exactly 98 die; rather, this refers to the mathematical concept of expectation of a random variable. The expectation for the number to die in a week is 98/52 that is, approximately 2 people.

b) What is the probability that a person celebrating their 22^{th} birthday will survive to reach 23?
The probability that a particular person dies during the year is 0.098%, or 0.0098; thus the probability that they do not die is 1-0.00098 or .99902; that is, approximately 99.9%

c) If we have a club of 100 people (each 22 years old), what is the probability that there will be at least one death among the members in the coming year?
The probability of no deaths among 100 people is $(0.99902)^{100}$; *that is 0.907 or 90.7%. Thus the probability of at least one death is 1-0.907 or .093.*

d) If we have a club of 100 people (each 20 years old), what is the probability that at least one member will still be alive after one year?
The probability that everyone dies is $(0.00098)^{100}$, *or which is less that* 10^{-300}, *which is so low that it is effectively 0. That is, the probability that least one member will still be alive is effectively 1.*

e) In a city with 5,000,000 people (of various ages), how many of them would be expected to die in a week?
*To answer this we need a mortality rate for a population of mixed ages. If we assume we can use as an estimate the overall rate for the whole US population in 2005, we can say the expected number of deaths is 825 per 100,000 or 0.825%. Thus in 5,000,000, the deaths per week is 0.00825*5000000/52 = 793.*

Q2. Suppose your car has five failure modes: wreck (MTTF=20yrs; MTTR = 2 wks), mechanical (1 yr; 3 wks), electrical (3 yrs; 1 day), flat-tire (3 yrs; 1 hr), out-of-gas (3 yrs; 5 hrs). What is the overall MTTF for the car? What is the availability? What would change in these measures if we introduced a warning system that completely prevented out-of-gas from occurring?

In a 1 yr period, we expect 1/20 = 0.05 wreck, 1 mechanical failure, 1/3 =0.33 electrical failure, 1/3=0.33 flat-tire, and 1/3=0.33 out-of-gas event. In all, we expect 2.05 failures in a year, so the AFR is 2.05, and the MTTF is thus 1/2.05 yr, or about 6 months.

*Consider each failure mode as the failure of a component of the car; the system only works if all components are functioning. As we are given MTTF and MTTR, the availability of each mode is MTTF/(MTTF+MTTR); but we need to work in consistent units, which I will take as years. So for wreck, MTTR is 2/52=0.04 yrs, so availability is 20/(20+0.04)=.998. For mechanical, MTTR is 0.06 yrs, so avail=1/(1.06)=0.943. For electrical, MTTR is 1/365=0.003 yr, so avail = 3/3.003=0.999. For tire, MTTR = 1/(365*24)=0.00011 yr, so avail = 3/3.00011=0.99996. For out-of-gas, MTTR is 5/(365*24)=0.0006, and avail = 3/(3.0006)= 0.9998. Thus the availability of the whole car is the product of these availabilities, that is, 0.998*0.943*0.999*0.99996*0.9998 = 0.94. (It wasn't asked, but one can then find overall MTTR from overall MTTF and overall availability!)*

*If we prevent out-of-gas events, we repeat the calculation without this component, giving MTTF= 1/(0.05+1+0.33+0.33)=0.58 yr, and avail is 0.998*0.943*0.999*0.99996 which is again about 0.94.*

Q3. Suppose we have a particular computer system which cost us $10,000. The system has AFR=5% (that is, we expect only 0.05 failures per year of operation), and this system can be used to process a type of job which takes 1 hr to run. The job brings $1 profit to us each time it runs successfully, but we have to pay compensation of $100 to the user if the system fails during the job. What is our expected financial return from our purchase of the system? Is it financially sensible to buy a second system and set up a redundant 2-plex? How might some of the parameters change, to lead you to alter your conclusion?

Note: the question does not say that there is a chance of 0.05 of failure in a single job; this is the expectation of the number of failures during the whole year!

A key fact we need, which isn't stated, is the availability of the system; this depends on the MTTR, since we can work out MTTF from the AFR. The MTTF is 20 yrs, let's take a fairly extreme assumption that MTTR is 1 yr. Then availability of the system is 20/(20+1)=0.952.

*Another key fact is how many jobs are available; that is, how busy will the system be? Let us suppose initially that we have enough work to keep it busy all the time; that is, there are at least 24*365 jobs that we would be paid to process, per year.*

*For the system as described, with the assumptions given, we would expect to do jobs whenever the system is available. That is, during one year, we would do 0.952*24*365=8343 jobs, bringing $8343 profit. We would expect only 0.05 of a failure, for expected compensation payout of 0.05*100=$5 during the year. Thus we expect a net return of $8343-5=$8337 during one year, from an outlay of $10000.*

If we instead spend $20,000 so we have it as a redundant 2-plex, we would improve availability to $1-((1-0.952)^2)=0.9977$. *MTTF(2-plex) would be* $MTTF(simple)^2/2*MTTR(simple) = 20^2/2*1 = 200$, *and AFR(2-plex) would be 0.005. Thus during a year, profit would be 0.9977*24*365 = $8740, and compensation would* *be 0.005*100 = $0.50, for net return of $8740. Thus the extra investment of $10,000 increases our return by only $403. This is not a good investment. If there were even a few more jobs to do than saturates one system, we would do well to use the second system to run other jobs, instead of, or as well as, being a backup for the main system! The value of a redundant 2-plex comes when the penalty for failure is much higher: if each failure required compensation of $1,000,000, then without a 2-plex, we would face a net loss, but with a 2-plex there is a small net return.*