Task:
a) Show that the derivative for the hyperbolic tangent sigmoid:

$$a = \frac{e^{n} - e^{-n}}{e^{n} + e^{-n}}$$

is $1-a^2$
b) Write the backpropagation rule for the hyperbolic sigmoid transfer function

Solution:

a) The goal of this exercise is to show that there is a convenient way for computing the derivative of the hyperbolic tangent sigmoid similarly to the sigmoid function where the derivative was a(1-a), a=f(n).

$$f(n) = a = \frac{e^{n} - e^{-n}}{e^{n} + e^{-n}}$$

$$\frac{\partial f(n)}{\partial n} = \frac{\partial}{\partial n}\left(\frac{e^{n} - e^{-n}}{e^{n} + e^{-n}}\right) = \frac{\left(e^{n} - e^{-n}\right)'(e^{n} + e^{-n}) - \left(e^{n} - e^{-n}\right)(e^{n} + e^{-n})'}{(e^{n} + e^{-n})^2} =$$

$$= \frac{\left(e^{n} - (-1)e^{-n}\right)(e^{n} + e^{-n}) - \left(e^{n} - e^{-n}\right)(e^{n} + (-1)e^{-n})}{(e^{n} + e^{-n})^2} = \frac{(e^{n} + e^{-n})^2 - \left(e^{n} - e^{-n}\right)^2}{(e^{n} + e^{-n})^2} =$$

$$= 1 - \frac{\left(e^{n} - e^{-n}\right)^2}{(e^{n} + e^{-n})^2} = 1 - a^2$$

b) for output neurons:

$$\delta_i^{\zeta} = \left(d_i^{\zeta} - o_i^{\zeta}\right) \cdot f'\left(net_i^{\zeta}\right) = \left(d_i^{\zeta} - o_i^{\zeta}\right) \cdot (1 - o_i^{2\zeta})$$

$$\Delta w_{ji} = \eta \cdot \sum_{\zeta} \delta_i^{\zeta} \cdot o_j^{\zeta} = \eta \cdot \sum_{\zeta} \left(d_i^{\zeta} - o_i^{\zeta}\right) \cdot (1 - o_i^{2\zeta}) \cdot o_j^{\zeta}$$

for hidden neurons:

$$\delta_j^{\zeta} = f'(net_j^{\xi}) \cdot \sum_i w_{ji} \cdot \delta_i^{\zeta} \cdot = (1 - o_j^{2\xi}) \cdot \sum_i w_{ji} \cdot \delta_i^{\zeta}$$