Numerical Computation of Differential-Algebraic Equations for Nonlinear Dynamics of Multibody Android Systems in Automobile Crash Simulation

Bud Fox, Leslie S. Jennings, and Albert Y. Zomaya,* Senior Member, IEEE

Abstract—The principle of virtual work is used to derive the Euler–Lagrange equations of motion in order to describe the dynamics of multibody android systems. The constrained variational equations are in fact differential-algebraic equations of high index and are cast as ordinary differential equations through differentiation of the constraint equations. The integration routine LSODAR and the fourth-order Runge–Kutta method are used to compute the generalized coordinates, their time derivatives and the body forces of two android models. The graphs of the constraint forces reveal the whiplash effect on the neck and that the stiffness of both multibody systems is due to large magnitude impulsive forces experienced by many bodies simultaneously.

Index Terms—Automobile crash simulation, constrained variational systems, differential-algebraic equations, mathematical modeling, multibody dynamics, nonlinear contact forces, virtual work.

I. INTRODUCTION

MULTIBODY dynamics and constrained variational problems have attracted the attention of many scientists and the need for increased safety in the automotive environment demands more knowledge of android multibody system behavior in automobile accidents involving spinal and head injuries. The dynamic analysis of multibody systems is aided by various software packages, for example, automatic dynamic analysis of mechanical systems (ADAMS)1 [14], dynamic analysis and design system (DADS) [18] and mathematical dynamical models (MADYMO) [19], which has been used extensively for automobile/android accident simulation; the ADAMS software has various components for modeling specific dynamical systems including ADAMS/Android. Here the authors use their own developed general multibody software, Multibody System, to model two multibody android systems subject to various conditions.

The study of constrained variational equations arising from multibody dynamics and differential algebraic equations (DAE’s), is related, due to the augmentation of the differential equations with algebraic constraints. Simeon et al. [17] reviews the theory and computation of DAE’s, nilpotency and index, initial value problems, differential geometry, numerical methods, and applications to multibody dynamics. Brenan et al. [2] provide a thorough introduction to the types of DAE’s, constrained variational problems, the theory, solvability and index concept, numerical methods and software for their computation, and examples of DAE’s in multibody systems concerning: rigid body systems, trajectory control, electrical networks, and the method of lines.

Automobile crash simulation involving two planar android multibody system models is investigated. The first model is a simplified 11 body-part android system showing the general motion and forces on large body-part clusters. The second model is a 46 body-part android muscle-skeletal system incorporating the cervical, thoracic, lumbar and coccyx vertebrae and their pseudomuscle restoring torques. Thoracic and lumbar restraining forces (provided by seat belts) are used in both models, and body contact forces between the body parts and the seat, and body-restoring torques, use an appropriate selection of spring and damping coefficients reflecting overdamping in their respective differential equation representations. The second android model experienced more nonlinear oscillatory forces which results in very stiff numerical behavior of the dynamic system. Recently Petzold [20] has published work on a similarly behaved highly oscillatory constrained multibody dynamic system and explores methods to obtain increased numerical solution convergence using a successful coordinate-split (CS) technique. The simulation duration for forward collisions is determined by the time for which the crash impulse takes place; time spans of approximately 0.20 s of real time suffice for dynamic analysis.

Many researchers provide models whose coordinate systems are difficult to interpret and/or the software for system equation computation is not easily examinable or available in the public domain. This leads to a difficulty to reproduce research results provided by authors for the purpose of comparison of efficiency and accuracy of computational methods. Here, generalized planar Cartesian coordinates are used and the software titled Multibody System uses commonly available subroutines found in LAPACK, LINPACK, and the LSODAR numerical integrator, for ease of result reproduction and comparison.
The constrained variational android equations form a system of DAE’s and is transformed to a system of ODE’s which is computed using the fourth-order Runge–Kutta method and the subroutine LSODAR provided by Petzold and Hindmarsh [12] (see also [2]). Other studies concerning DAE’s have been addressed in [1], [5], [9]–[11], [13], and [20]. Step size selection, method switching, singularity occurrence, number of function evaluations and additional information are produced by LSODAR for comparing efficiency and accuracy of the numerical computation of multibody system equations.

The rest of the paper is divided as follows. Section II provides the derivation of the multibody system equations using the principle of virtual work. Section III shows how an index three DAE may be cast as an underlying ODE. Section IV introduces the android multibody system. Section V presents the numerical results, animation and accompanying computational difficulties and finally Section VI reviews the researched ideas.

II. MULTIBODY SYSTEM EQUATIONS

The general equations of dynamic equilibrium for multibody systems can be formulated using generalized Cartesian coordinates and the principle of virtual work: the derivation follows closely the work of Shabana [16]. This principle states that the virtual work due to the inertia forces is equal to the sum of the virtual work due to the externally applied forces and constraint forces. That is

\[ \delta W_{I,i} = \delta W_{e,i} + \delta W_{c,i}. \]  

(1)

The virtual work of the externally applied forces acting on the rigid body \( i \) is

\[ \delta W_{e,i} = \mathbf{Q}_{e}^{T} \delta \mathbf{q}_i. \]  

(2)

where \( \mathbf{Q}_{e,i} \) is the vector of generalized externally applied forces corresponding to the vector of generalized planar coordinates

\[ \mathbf{q}_i = [R_{i}, \theta_{i}]^{T}. \]  

(3)

The virtual work of the joint constraint forces acting on the rigid body is

\[ \delta W_{c,i} = \mathbf{Q}_{c}^{T} \delta \mathbf{q}_i. \]  

(4)

where \( \mathbf{Q}_{c,i} \) is the vector of generalized constraint forces corresponding to the generalized coordinates \( \mathbf{q}_i \). Finally, the virtual work due to the inertia forces is given as

\[ \delta W_{I,i} = \int V_i \rho_i \mathbf{r}_i^{T} \delta \mathbf{r}_i \, dV_i. \]  

(5)

where \( \rho_i \) and \( V_i \) are the density and volume of body \( i \), respectively, the position of an arbitrary point on the body \( i \) with respect to the global coordinate system is

\[ \mathbf{r}_i = \mathbf{R}_i + \mathbf{A}_i \mathbf{u}_i. \]  

(6)

\[ \Rightarrow \dot{\mathbf{r}}_i = \dot{\mathbf{R}}_i + \frac{d}{d\theta_i} \mathbf{A}_i \mathbf{u}_i \]  

(7)

and the rotation matrix of the body \( i \) with angle of rotation \( \theta_i \) is

\[ \mathbf{A}_i = \begin{bmatrix} \cos \theta_i & -\sin \theta_i \\ \sin \theta_i & \cos \theta_i \end{bmatrix}. \]

The virtual work of the inertia forces of body \( i \) may be expressed as

\[ \delta W_{I,i} = (\mathbf{M}_i \dot{\mathbf{q}}_i - \mathbf{Q}_{e,i})^{T} \delta \mathbf{q}_i. \]  

(8)

where \( \mathbf{Q}_{e,i} \) is the vector of centrifugal inertia forces. On substitution of (2), (4), and (8) into (1) one obtains

\[ (\mathbf{M}_i \dot{\mathbf{q}}_i - \mathbf{Q}_{e,i} - \mathbf{Q}_{c,i})^{T} \delta \mathbf{q}_i = 0. \]  

(9)

It may be shown that the system equations are

\[ \mathbf{M} \dot{\mathbf{q}} + \mathbf{C}_q^{T} \lambda = \mathbf{Q}_e. \]  

(10)

which represent a system of \( 3NB \) equations in \( 3NB + NC \) unknowns; \( NB \) is the total number of bodies in the planar system and \( NC \) is the total number of independent constraint equations. Since there are more unknowns than equations, the constraint equations representing the constrained motion of the system, are required to be adjoined to the system equations. The constraint equations denoting body connectivity may be written as

\[ \mathbf{C}(\mathbf{q}, t) = 0. \]  

(11)

and upon differentiating twice with respect to time gives

\[ \mathbf{C} \ddot{\mathbf{q}} = \mathbf{Q}_c. \]  

(12)

Augmenting the system equations (10) with (12) one obtains the complete system of \( 3NB + NC \) equations in \( 3NB + NC \) unknowns

\[ \begin{bmatrix} \mathbf{M} & \mathbf{C}_q^{T} \\
\mathbf{0} & \mathbf{C}_q \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}} \\
\lambda \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_e \\
\mathbf{Q}_c \end{bmatrix}. \]  

(13)

III. DIFFERENTIAL ALGEBRAIC EQUATIONS

Differential algebraic equations or DAE’s, are differential equations augmented with algebraic constraints involving variables for which the system is to be computed. Petzold and co-authors, have published material on the solvability and computation of DAE’s, see [2] for an introduction to DAE’s and references therein concerning the earlier work on the theory and computation, and also [1], [3]–[5], [9]–[11], [13], [17], and [20]. A DAE may be of the form

\[ F(\mathbf{x}, \dot{\mathbf{x}}, t) = 0. \]  

(14)

The DAE of the multibody system considered here is of the form

\[ \begin{bmatrix} \mathbf{I} & 0 & 0 \\
\mathbf{0} & \frac{\partial \mathbf{C}}{\partial \mathbf{q}} & 0 \\
\mathbf{0} & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}} \\
\dot{\mathbf{v}} \\
\dot{\lambda} \end{bmatrix} = \mathbf{f}(\mathbf{q}, t) = \begin{bmatrix} \mathbf{v} \\
\mathbf{Q}_c \\
\mathbf{C}(\mathbf{q}, t) \end{bmatrix}. \]  

(15)

where \( \dot{\lambda} = \lambda \) and \( \dot{\mathbf{v}} = \dot{\mathbf{q}}. \) The matrix on the left hand side is however singular, differentiating the third equation above with respect to time twice, yields

\[ \begin{bmatrix} \mathbf{I} & 0 & 0 \\
\mathbf{0} & \frac{\partial \mathbf{C}}{\partial \mathbf{q}} & 0 \\
\mathbf{0} & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}} \\
\dot{\mathbf{v}} \\
\dot{\lambda} \end{bmatrix} = \mathbf{f}(\mathbf{q}, \dot{\mathbf{q}}, t) = \mathbf{f}(\mathbf{q}, \mathbf{v}, t). \]  

(16)
which has a nonsingular leading matrix providing $\partial C/\partial q$ is of full rank for all time. The following definition classifies a DAE with respect to differentiation of the system equations given by (14).

Definition: The minimum number of times that all or part of the DAE $F(x, x, t) = 0$ must be differentiated with respect to $t$ in order to determine $x$ as a continuous function of $x$ and $t$, for $t$ in some interval, is the index of the DAE [2].

The original system has been differentiated twice and the substitution of $\dot{\mu} = \lambda$ can be considered as an additional differentiation. This results in the ODE shown in (16) and hence the original system (15) is regarded as a DAE of index three. Note that here

$$\mu(t) = \int_0^t \lambda(\tau) \, d\tau$$

is computed by the ODE software. To find $\lambda, \mu(t)$ must be differentiated, this is, however, an unstable process. Petzold et al. [2], discuss the computational/numerical difficulties that may arise as a result of differentiating the constraint equations; the constraint equations may not be satisfied as the integration progresses and excessive differentiation of the constraints is not recommended.

IV. MULTIBODY MODELS

Two android multibody systems are investigated in order to determine the generalized coordinates, their time derivatives, body forces and body torques for automobile crash simulation. The simplified model involves generalized body-part clusters, whereas the vertebral model concerns all vertebrae, ribs and general structure for spinal injury analysis.

A. Simplified Body Cluster Model

The simplified body cluster multibody system has a total of 13 bodies, 11 of which are android body-parts representing clusters of smaller bodies. These parts are: head, neck, thorax, lumbar, bottom, thigh, lower leg, foot, upper arm, lower arm and hand. The force models required are: body-seat, foot-floor, seat belt, and restoring torque, of which the first two are documented here; the latter two are elementary spring-damper/torque models.

1) Body Seat Contact Force Model: This model involves the interaction of a pseudobody-part in contact with the seat. Damped, oscillatory motion is observed through this form of contact model, where the spring and damping coefficients may be changed to simulate differing seat support compounds.

The contact force model used for the body-seat interaction employs a spring-damper element between points $p_i$ and $p_j$, the former residing on the seat surface and the latter on the most penetrated portion of the body-part. The magnitude of this force is

$$F_{cij} = k\delta_{ij} + c\dot{\delta}_{ij}$$  \hspace{1cm} (18)

where $k$ and $c$ are the spring and damping coefficients respectively and

$$\delta_{ij} = R_{y_{ij}} - R_{p_{ij}} - \left(r + \frac{l_2}{2}\right), \hspace{1cm} (19)$$

Shabana [16] states that the virtual work due to a force with magnitude $F_{cij}$ is

$$\delta W_{ij} = F_{cij} \frac{\partial r_{p_{ij}}}{\partial \delta_i} \left[ \frac{\partial r_{p_{ij}}}{\partial \delta_i} \right] \delta q_i$$

$$= Q_i^T \delta q_i + Q_j^T \delta q_j \hspace{1cm} (20)$$

where

$$Q_i = [Q_{R_{ij}} \ Q_{\theta_{ij}}] = F_{cij} \left[ \frac{\partial T_{i}}{\partial \theta_i} \frac{\partial T_{i}}{\partial \theta_i} \right] \frac{r_{p_{ij}}}{|r_{p_{ij}}|} \hspace{1cm} (22)$$

$$Q_j = [Q_{R_{ij}} \ Q_{\theta_{ij}}] = -F_{cij} \left[ \frac{\partial T_{j}}{\partial \theta_j} \frac{\partial T_{j}}{\partial \theta_j} \right] \frac{r_{p_{ij}}}{|r_{p_{ij}}|} \hspace{1cm} (23)$$

and the position vector written in the global coordinate system is

$$r_{p_{ij}} = r_{p_{i}} - r_{p_{j}} = R_i + A_i \bar{u}_{p_{i}} - R_j - A_j \bar{u}_{p_{j}}. \hspace{1cm} (24)$$

To determine the direction along which the force acts, the global position vector given by (24), of the spring-damper element, needs to be determined. The global position vector of the origin of the body $j$ coordinate system with respect to that of the body $i$ coordinate system is

$$\bar{u}_{ij} = R_j - R_i. \hspace{1cm} (25)$$

This may also be written as

$$\bar{u}_{ij} = A_i \bar{u}_{O_{j}i}$$

$$\Rightarrow \bar{u}_{O_{j}i} = A_j^T \bar{u}_{ij}. \hspace{1cm} (26)$$

The two contact criteria used are $-(l_1/2) \leq \bar{u}_{O_{j}i} \leq (l_1/2)$ and $\delta_{ij} = \bar{u}_{O_{j}i} - (r + (l_2/2)) \leq 0$, where the $i$th coordinate system is located at the centroid (middle in this case) of the seat. The local position vector of the point $p_i$ with respect to the body $i$ frame is

$$\bar{u}_{p_{i}} = \left[ \frac{\bar{u}_{O_{j}i} \bar{r}_{i}}{l_2} \frac{\bar{r}_{i}}{2} \right]. \hspace{1cm} (27)$$

The local position vector of the point $p_j$ with respect to the body $j$ frame (note, $\bar{u}_{p_{ij}} = \bar{u}_{p_{j}}$), is found as follows:

$$R_i + A_i \bar{u}_{p_{ij}} = R_j + A_j \bar{u}_{p_{j}}$$

$$\Rightarrow \bar{u}_{p_{ij}} = A_j^T (R_i - R_j + A_i \bar{u}_{p_{ij}})$$

$$= A_j^T (R_i - R_j + A_i \left[ \frac{\bar{u}_{O_{j}i} \bar{r}_{i}}{l_2} \right]) \hspace{1cm} (28)$$

Now the vector $r_{p_{ij}}$ may be determined and hence the virtual work due to the contact force may be computed.

Result Reproduction

Scientists aiming to reproduce results should be aware of sign convention and appropriate selection of spring and damping constants $k$ and $c$, respectively. If $F_{cij} > 0$ then the restoring force acts in the direction of $r_{p_{ij}}$, if however, $F_{cij} < 0$, then the restoring force acts in the opposite direction to $r_{p_{ij}}$. For example, if $F_{cij} < 0$, then

$$Q_i = [Q_{R_{ij}} \ Q_{\theta_{ij}}] = \left[ \begin{array}{c} <0 \\ <0 \end{array} \right] \hspace{1cm} (29)$$
and the force on body $i$ in the global $y$ direction will be downwards, in the opposite direction to $\mathbf{r}_{p_{ij}}$ as one would expect if a body were pushing on the seat supporting it. Newton’s Third Law implies that an equal and opposite force would be exerted on the body $j$, effectively pushing it upwards in this case. If $x = \delta_{ij}$ and $\mathbf{F}_{c_{ij}} = -k\delta_{ij} - c\dot{\delta}_{ij}$ then the second-order linear ODE

$$m\ddot{x} + c\dot{x} + kx = 0$$

(30)

results. In the case where two distinct real roots and hence over damping is desired, the appropriate condition for selection of $k$ and $c$, where $k > 0$ and $c > 0$ is

$$0 < \frac{4km}{c^2} < 1.$$ 

(31)

In the case where two complex roots and hence under damping is desired, the appropriate condition for selection of $k$ and $c$, where $k > 0$ and $c > 0$ is

$$1 < \frac{4km}{c^2}.$$ 

(32)

Since the contact force given by (18) is highly impulsive in nature due to the nonzero $\dot{\delta}_{ij}$ when contact occurs, an alternative progressive pseudodamping constant $-\alpha\dot{\delta}_{ij}$, is used, and the resulting force is

$$\mathbf{F}_{c_{ij}} = k\delta_{ij} - \alpha\dot{\delta}_{ij}\delta_{ij}$$

(33)

the negative sign is required since $\dot{\delta}_{ij} < 0$ for the contact duration. This would seem to be a better model of two “soft” bodies coming in contact with nonzero velocity.

2) Foot Floor Contact Force Model: This model involves the contact of an android foot with the car floor. A similar derivation of the contact force may be used for both the heel and the front ball of the foot. Although only two contact points are used here, it is possible to extend this idea and use more contact points allowing improved weight distribution over the shoe.

The derivation of the contact force is as in the above body seat contact force model. Here, the local position vectors of interest to determine the vector $\mathbf{r}_{p_{ij}}$ are

$$\mathbf{u}_{p_{i}} = \left[ \begin{array}{c} \frac{l_{1} \cos \theta_{i}}{2} \\ \frac{l_{1} \sin \theta_{i}}{2} \end{array} \right] + \left[ \begin{array}{c} r \cos \left( \frac{3\pi}{2} - \theta_{i} \right) \\ r \sin \left( \frac{3\pi}{2} - \theta_{i} \right) \end{array} \right]$$

$$\mathbf{u}_{p_{j}} = \left[ \begin{array}{c} \frac{l_{2} \cos \theta_{i}}{2} \\ \frac{l_{2} \sin \theta_{i}}{2} \end{array} \right]$$

where

$$\mathbf{r}_{B} = \mathbf{R}_{j} + \mathbf{r}_{B_{ij}} = \mathbf{R}_{j} + A_{j}\mathbf{u}_{B_{ij}}$$

$$\Rightarrow \mathbf{u}_{B_{ij}} = A_{j}^{-1}\left( \mathbf{r}_{B} - \mathbf{R}_{j} \right).$$

Note that also

$$\mathbf{r}_{B} = \mathbf{R}_{j} + A_{j}\mathbf{u}_{B_{ij}}.$$

The amount of penetration of body $i$, where $r$ is the radius of the heel and ball of the foot, is

$$\delta_{ij} = r_{B_{ij}} - r - \left( R_{j} - \frac{l_{2}}{2} \right)$$

now one may obtain $\dot{\delta}_{ij}$ and the final contact force. A similar procedure is used to determine the local position vectors for the front ball of the foot and, hence, $\mathbf{r}_{p_{ij}}$, the contact force $\mathbf{F}_{c_{ij}}$ and finally the generalized force vectors $\mathbf{Q}_{B}$ and $\mathbf{Q}_{j}$. The seat belt, restoring torque and shoulder connectivity models use similar physical derivations to the models described above.

B. Vertebral Body Model

The vertebral multibody system has a total of 48 bodies, 46 of which are android body-parts, that is: head, six cervical vertebrae, 12 thoracic vertebrae, five lumbar vertebrae, one coccyx element, bottom, thigh, lower leg, foot, upper arm, lower arm, hand, shoulder plate, clavicle, and 12 ribs. The force models required are those of the simplified model and also a shoulder connection model representing muscle-skeletal tissue in the form of spring and damping forces and revolute joints. The vertebral column is modeled using revolute joints and restoring torques, as is the rib cage and much of the remaining muscle-skeletal system. Figs. 1 and 2 clearly illustrate the connectivity of the two android models discussed. The structure of the constraint equations represents the connectivity of the chain-like multibody system.

V. RESULTS

The vehicle crash simulations involve: two android models composed of 11 and 46 body parts, seat belts restraining the waist and/or thorax, front impact force, and initial configurations given by constraint equations. The initial conditions of both systems are such that the bottom of the android is at the point of contact with the seat and all bodies have zero velocity prior to the front impact. The generalized coordinates $q_{i}$ are not graphed since the positions and orientations may be observed by the eight slide, 0.025 s interval, animations shown in Figs. 1, and 2. However, the generalized external and constraint forces, $\mathbf{Q}_{B}$ and $\mathbf{Q}_{j}$, respectively, are plotted for the impact durations shown in Figs. 3–5.

A. Result Generation

The selection of spring and damping coefficients for the linear and torsional spring-damper elements as discussed in Section IV-A1 and the choice of crash duration time and integrator time step size is central to the comparison of model behavior. Young [15] indicates that the impulse $J$ of a force $\mathbf{F}$ in a time interval $t_2 - t_1$ is

$$J = \int_{t_1}^{t_2} \mathbf{F} \, dt = \int_{v_1}^{v_2} m \, dv$$

(34)

hence

$$\mathbf{F} = \frac{m(v_2 - v_1)}{t_2 - t_1}.$$ 

(35)
For a mass range of 50 kg to 100 kg, a crash duration of 0.1–0.2 s and a change in velocity of between 11 and 17 ms$^{-1}$, the crash velocities can be considered to range between 40 and 60 kmh$^{-1}$ and the impact forces between 2000 and 17 000 N. Naturally for a fixed mass, the smaller the change in velocity and the longer the impact duration, the smaller the force of impact. Shown in Figs. 1 and 2 are the generalized cases of front crash with both waist and thoracic belts (simplified and vertebral models) for an impact force of approximately 15 000 N, or equivalently for a change of velocity to 40–60 kmh$^{-1}$ from 0 kmh$^{-1}$.

The time step sizes for the Euler and Runge–Kutta (fourth-order) numerical integrators of both models are chosen according to the frequency of oscillation, or rate of damping of a typical second-order differential equation of the form

$$m\ddot{x} + c\dot{x} + kx = 0. \quad (36)$$

In order that the integrator be able to follow the system oscillation, the time step size should be smaller than the reciprocal of the natural frequency, given as

$$f = \sqrt{\frac{c^2}{4m^2} + \frac{k}{m}}. \quad (37)$$

if $c^2 < 4$ km, or smaller than the reciprocal of the largest exponential damping coefficient, which is of the order of $c/m$ if $c^2 > 4$ km.

The simplified model uses the integration routine LSO-DAR with relative (RTOL) and absolute (ATOL) tolerance parameters set to $10^{-5}$. The routine switches between nonstiff and stiff regions of dynamic behavior and uses the Adams and implicit methods, respectively. One observes the equal and opposite nature of the $Q_{car}, Q_{car}$, and $Q_{car}, Q_{car}$ graphs (Figs. 3 and 4) concerning the seat-base and seat-back, this
is due to the seat-base and seat-back being fixed relative to each other. The nondifferentiable nature of $Q_{cyl}$ of the seat-back is due to a combination of body contact and seat belt forces acting simultaneously on the seat and thorax and bottom bodies.

The more complete vertebral model uses the fourth-order Runge–Kutta integration scheme with step size $h = 10^{-4}$, since LSODAR experienced difficulty in integrating the ODE due to the excessive stiffness present in the collaborative dynamics of the contact force models governing the right hand side. This means that LSODAR knows that the trajectories cannot be followed to the accuracy required, whereas the Runge–Kutta method does no testing of accuracy. Figs. 4 and 5 indicate that the head, thorax, abdomen and all cervical vertebrae C1–C6, experience an increase in the generalized body constraint force prior to their increase since the thorax and lumbar seat belts tug backward and downwards on the body and these forces are propagated throughout. The rotative whiplash effect is visible in the vertebral model where the joint torques $Q_{\theta}$ of the head and neck all undergo clear magnitude and sign changes. The vertebral model captures the dynamic behavior of the simplified model but with more complete detail. The nondifferentiable corner on the graphs of Fig. 5 at approximately 0.02 s is due to the head being forced out of the back of the seat as described in the contact force model of Section IV-A1.

The accuracy of the results was determined by checking the satisfaction of the following inequality

$$|\mathbf{C}(\mathbf{q}, t) - \mathbf{C}(\mathbf{q}, 0)| < h$$

where $h$ is the integrator time step size. The LSODAR routine allowed the simplified android model to satisfy the inequality for the entire simulation time, but the Runge–Kutta method satisfied this criterion until $t = 0.1115$; by the end of the simulation the left hand side of the inequality had the value 0.00018, which is marginally greater than the time step size of 0.0001. Note that the normal accuracy expected of Runge–Kutta (namely $h^4 = 10^{-16}$), cannot be attained here because the right hand side of the ODE does not have continuous derivatives, see (33). It needs to be differentiable to fifth order.

Wismans [19] discusses the evolution of the MADYMO software package for crash simulation which incorporates FEM techniques, force interaction models, six injury parameter calculations including Head Injury Criterion (HIC), Gadd
Severity Index (GSI), 3-ms Criterion (3MS), Thoracic Trauma Index (TTI), Viscous Injury Response (VIC) and Axial loads, seat belt and air bag models, crash dummy databases and applications including thorax side impact simulation, unrestrained truck driver simulation and an accident involving a truck and a cyclist. They introduce Generator of Body Data (GEBOD) which is a public domain program that provides geometric and inertial properties of human beings composed of 15 segments. Here the physical data required for android modeling, including mass, moments of inertia and dimensions of body parts, has been taken from [6], and the appropriate number of body parts has been explored using the aforementioned simplified and vertebral models.

Huston [7] and [8], discuss briefly some methods of automobile crash simulation and multibody modeling; the former introduces body-part injury assessment formula such as HIC and SI, but indicates their reported inaccuracy in predicting injury, and the latter presents an idea analogous to the method of component mode synthesis of structural analysis, explored by many authors. This method involves the grouping of bodies of similar physical properties to avoid convergence problems in the integration of the ODEs describing the system. Huston indicates that bodies of smaller masses may be isolated and analyzed separately and the constraint equations can combine the dynamic behavior of the other bodies of larger mass. This approach is not taken here and the constraint equations used provide information on body-part connectivity. Stiffness in the system equations may be a combination of not only the differing physical properties of the constituent bodies but also the contact force and joint restoring torque models which are dependent on linear or torsional spring-damper elements. Adjusting these models may provide smoother contact force models that are easier to integrate but may not represent the true physical behavior of the dynamic system.

Although Figs. 3–5 show only a subset of the body forces, the complete set has been generated (not shown); those presented are considered of greater interest. Their purpose is to indicate the changing magnitude and direction of force experienced by the individual body part and any stiff behavior in the body dynamics due to bending moments and axial or shear forces experienced by the body.

VI. CONCLUSIONS

The Euler–Lagrange equations were used to construct high index differential algebraic system equations for two android multibody models. The DAE’s of index three were transformed into DAE’s of index zero, or equivalently the underlying ODE’s. These were computed using the LSODAR integration routine and the fourth-order Runge–Kutta method. The results indicate that large magnitude body constraint forces are present and are the cause of stiffness in the android dynamical systems and present numerical integration difficulties. The contact force modeling uses a progressive damping force which is a function of both the body penetration distance and its time derivative. The simplified model predicts largely the behavior of the body dynamics, however, the vertebral model reveals greater constituent body-part detail including the whiplash effect on the cervical vertebrae. Further work will involve extending the models from the plane into three dimensions and exploring more accurate methods of numerical integration.

REFERENCES


Bud Fox received the B.Sc. degree, with honours, in applied mathematics from The University of Western Australia, Perth, Australia, in 1996. Currently he is located at The University of Western Australia, pursuing the Ph.D. degree majoring in applied mathematics and computational science. His Ph.D. work concerns multibody dynamics, differential algebraic equations, Lagrange’s equations, and mathematical modelling. The computer code, titled Multibody System, has been written in an effort to merge the aforementioned disciplines.

Leslie S. Jennings was born in Adelaide, South Australia, on August 25, 1947. He received the B.Sc. (Honors) from the University of Adelaide, Adelaide, South Australia, in 1969 and the Ph.D. degree in numerical analysis from the Australian National University, Canberra, Australia, in 1973.

He received a nine-month post-doctoral appointment to the Computer Science Department, Stanford University, Stanford, CA. Since then he has held a Lecturer, Senior Lecturer, then Associate Professor position in the Department of Mathematics at the University of Western Australia, Perth. His interests lie in numerical analysis, and in the application of optimal control to human movement modeling, multibody systems, chemical engineering and filter design. Currently, he is working on the interface of optimal control, numerical analysis, and software engineering.

Albert Y. Zomaya (S’88–M91–SM’97) received the Ph.D. degree from Sheffield University, Sheffield, U.K.

He is a Professor in the Department of Electrical and Electronic Engineering at the University of Western Australia, Perth, Australia, where he also leads the Parallel Computing Research Laboratory. He held visiting positions at Waterloo University, Waterloo, Ont. Canada, and The University of Missouri, Rolla. He is the author/co-author of more than 100 publications in technical journals, collaborative books, and conferences. He is currently an Associate Editor for the Journal of Parallel Algorithms and Applications, the Journal of Interconnection Networks, the International Journal on Parallel and Distributed Systems and Networks, and the Journal of Future Generation of Computer Systems. He previously served on the editorial boards of the International Journal in Computer Simulation and the IFAC Control Engineering Practice Journal. He is also the founding editor of the Wiley Book Series on Parallel and Distributed Computing. He is the author/co-author of four books and the editor of three volumes. He is the Editor-in-Chief of The Parallel and Distributed Computing Handbook (New York: McGraw Hill, 1996). He is the founding Co-Chair of the Workshop on Biologically Inspired Solutions to Parallel Processing Problems (BioSP3) (Florida, 1998; San Juan, 1999), the General Chair of the International Symposium on Parallel Architectures, Algorithms, and Networks (ISPAN) (Perth, Australia, 1999), the Program Vice-Chair of Second International Conference on Parallel and Distributed Computing and Networks (PDCC’98) (Brisbane, Australia, 1998), Vice-Chair of the 11th International Conference on Parallel and Distributed Computing Systems (Chicago, IL, 1998), and Vice Chair of the High-Performance Computing Conference (HiPC’97) (Banglore, India, 1997). He served on the steering and program committees of several national and international conferences. His research interests are in the areas of parallel algorithms, scheduling, computational machine learning, scientific computing, adaptive computing systems, mobile computing, and data mining.

Professor Zomaya is a board member of the International Federation of Automatic Control (IFAC) committee on Algorithms and Architectures for Real-Time Control, and serves on the executive committee of the IEEE Technical Committee on Parallel Processing. He is a chartered engineer and a member of the ACM, the Institute of Electrical Engineers (U.K.), and Sigma Xi. He received the 1997 Edgeworth David Medal from the Royal Society of New South Wales for outstanding contributions to Australian Science. He is currently an Associate Editor for the IEEE TRANSACTIONS ON PARALLEL AND DISTRIBUTED SYSTEMS, IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS (Parts A, B, and C).